

Maxwell Structures. 1

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Abstract

There exists a species of nonlinear system of exceptional relevance to behavioral scientists. It is the embodiment of a locally stable basin of attraction atop an unstable potential gradient. The universal importance of such para-stable entities was first identified by James Clerk Maxwell. This review characterizes the structures and the forces that can stabilize or destabilize them, and their relevance to us as psychologists and citizens.

Key words: balance-points, behavioral forces, dynamic systems, kindling.

Resumen

Existe una especie de sistema no lineal de excepcional importancia para los científicos del comportamiento. Es la encarnación de un nicho de atracción localmente estable encima de un gradiente de potencial inestable. La importancia universal de este tipo de entidades para-estables fue identificada por primera vez por James Clerk Maxwell. Esta revisión caracteriza las estructuras y las fuerzas que pueden estabilizar o desestabilizar dichas entidades y su importancia para nosotros como psicólogos y ciudadanos.

Palabras clave: puntos de equilibrio, fuerzas conductuales, sistemas dinámicos, ignición.

Systems are easiest to comprehend when their derivatives are linear functions of independent variables. But it is because of nonlinearities that systems such as ourselves exist, systems able to comprehend, and to write these reflective sentences.

Some nonlinearities are relatively tractable—simple quadratic and exponential functions, not far bent from straight and narrow. Others, edgy discontinuities, are essential adjuncts to comprehension, as they bound categories, separate entities, and characterize the binary logic of computers. Other nonlinearities, the enchanting changes of phase—from crystal to liquid, from laminar flow to turbulence, from matter to life—are intractable in the large, even when docile in microcosm. It is for these that the descriptor *emergent* so often emerges as explanation in lieu of explanation.

The success of physical sciences counsels the behavioral sciences to start simple. But not to start with the first simple thing found; instead to survey the domain to be essayed, so that easy things are chosen to lead, in progressive steps, to a successful assault on the more difficult. Maxwell structures provide the first simple interesting nonlinearity, and thus the subject of this essay.

Maxwell, the great Victorian physicist who unified the study of electricity and magnetism, and whose field equations remain among the most elegant in science, identified a nonlinearity of special interest:

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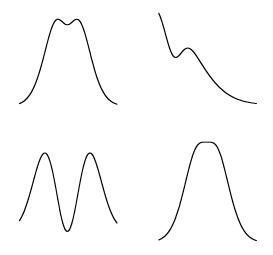
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"The rock loosed by frost and balanced on a singular point of the mountain-side, the little spark which kindles the great forest, the little word which sets the world a fighting, the little scruple which prevents a man from doing his will, the little spore which blights all the potatoes, the little gemmule which makes us philosophers or idiots. Every existence above a certain rank has its singular points: the higher the rank, the more of them. At these points, influences whose physical magnitude is too small to be taken account of by a finite being, may produce results of the greatest importance. All great results produced by human endeavor depend on taking advantage of these singular states when they occur." (1882/1969, p. 443)

Figure 1 represents these phenomena as *Maxwell structures*, sketched as a 2-dimensional projection of a multidimensional potential function. Following Maxwell, I use this particular nonlinear force function to characterize personal and social forces with quite different carriers than gravity and electromagnetism (Killeen, 1992). Laplace studied potential functions long before, but it is Maxwell's attention to a special class of potential functions that captures our attention. (Other equally important potential functions are discussed by Kelso, 1995). The function that characterizes the prototypical Maxwell structure—called *structure* here in reference to both its mathematical description as a function, and its embodiment in a physical or psychological entity—is the shallow potential well atop a large potential hill. The terrain along the other unpictured dimensions may take any shape, but must be locally uphill, or the system would roll out of this basin, as off a saddle, through the front or back of the page. It requires work to destabilize structures that reside in these high basins, just as it does to kindle a fire; but the energy then released is out of proportion to that work. Here I review the nature of these functions, the feedback systems which maintain or destroy them, and techniques that, to weal or woe, "take advantage of these singular states when they occur".

Figure 1. Clockwise from top left: A prototypical *Maxwell Structure*; an asymmetric one; an unstable one; an adamant one.



Ordinates

The x-axis of the Maxwell structures shown in Figure 1 are the values of a system parameter such as heat. It takes energy to make a spark; but if its heat carries kindling over a flashpoint, the energy is returned with compound interest. Maxwell structures may be symmetric, or may reside as concave ledges on an otherwise monotonic slope. This is the case for the spark: Only increases in temperature can move it over the lip.



The *y*-axis of Figure 1 is potential energy. This energy may be realized as work if the system is loosed to careen down the potential slope, transforming its elevation into kinetic energy. The forest contains much more potential energy before it is set after than after. The energy released may be squandered to the sky, or doled out in faggots to boilers and stoves for more useful work.

Ordered structures that inhabit the tops of potential hills are not more complex than those at the ground state, they are simpler. The description of matter in a healthy forest requires—and provides when received—less information than does its random bed of ashes. Statistical descriptions of such random ensembles are simple, but they are actually a retreat to a description of the class rather than of the individuals in it. Precise description of undifferentiated conglomerations at the ground requires voluminous, and worthless, information. Even Maxwell's demons cannot utilize that kind of information to perform work.

We may partition energy/enthalpy into bound energy and free energy; the former is a product of energy and temperature that is unavailable for doing work; the latter is the energy that may be converted into work, and is synonymous with negentropy. Whereas we think of nourishment as a way of gaining energy, Schrödinger (1944) observed that it was the greater negative entropy of the complex structured materials we consume compared to those that we excrete, that "fuels" life. It is far easier to convert an existing structure, such as a complex protein, to our own ends than to build one from the ground up. Metaphorically, we understand how some of the energy of other humans may be partially converted into work, but a portion always remains bound, not amenable to conversion.

Entropy and energy are closely related, both in thermodynamic terms, and in looser parlance. The generation of power requires compartments of matter at different temperatures that can be mixed. Although the total amount of energy remains the same after the mixture (the first law of thermodynamics), the entropy of the mixture is greater (the second law of thermodynamics). The free energy is less. The constituents are amenable to more economical description before the blending than is the mixture. Entropy concerns the partitioning of randomness, just as the analysis of variance concerns the partitioning of variance. We attach significance to structures that are sufficiently distinct from the ambient noise to justify the "degrees of freedom" lost in their separate categorization—that is, the naming of them. That discrimination can be achieved with a sufficiently broad structure even if it hugs the ground—in theory. Think of a small hill of energy—negentropy—a hill of structure/order in a flat desert. But more than a just-discriminable difference is necessary for the distinction to be useful. The higher the hill above the ground, the more effective the partition is in separating the energy levels, and the more informative is the resulting distinction. The child is admonished not to make a mountain out of a molehill. The greatest power is available when the proportion of variance that can be explained by the partition approaches 1.0; when the compartments start icy cold and burning hot, black and white, zero and one, towering and flat.

There is thus more that is knowable about ordered systems, those that have the potential for work, than about random ones. Once we understand the key to the system—have mastered its logic or its mathematics or its rhythm—we can predict and control it. Intrinsic order, and thus the potential for power, is a necessary condition for knowledge. Not all knowledge, however, is powerful. Some knowledge is merely significant, whereas other knowledge is potent. Knowledge is powerful if it corresponds to structures that rise high above the landscape of noise, permitting us to "take advantage of these singular states when they occur". In modern analysis this distinction is carried by the statistic called relative entropy, or Kullback-Leibler divergence. An unbiased estimate of this difference between distributions, measured in information theoretic units, is given by the Aikaike Information Criterion—a statistic seeing increasing use in model comparison to tell us how well the structure of our description matches the structure of the data. Rounding the circle, a century ago Henry Adams proposed a theory of history based



on the second law of thermodynamics; 50 years ago the MaxEnt school reinterpreted classical thermodynamics in information theoretic terms (e.g., Jaynes, 1957); and a generation ago, economic theory was interpreted in thermodynamic terms ("thermoeconomics"; e.g., Georgescu-Roegen, 1971). This hermeneutic cycle (Hermeneutic, 2014) is a helix that rises higher above the entropic plane as we learn more and more about the connectedness of theoretical structures; as each turn subsumes more under less. Thus, our knowledge of randomness and structure itself forms a potential structure.

Creating Maxwell Structures

Order out of chaos.

Because Maxwell structures are simpler than random ensembles, they are often created spontaneously wherever energy flows. Although the second law tells us that entropy is always increasing, it started minuscule, when all matter was concentrated, a universe of nothing and a dot of everything. The continual redistribution of energy since the big bang provides many opportunities for the evolution of locally organized structures, from Saturn's rings to the convection currents that roil the water of a teapot. The high-frequency energy received from the sun is the prime mover of life on earth; all of the rich structures we are and seek to understand are parasitic upon it, each finding their own way to do work by short-circuiting its ultimate reradiation to the night sky as lower frequency heat. These Maxwell structures emerge because they redistribute energy more efficiently than random processes (Prigogine & Stengers, 1984). While some are evanescent, others, like the red spot of Jupiter, persist for ages. An edge in an energy flow, shade in sun, fast among slow, are the provender for a feast on lambent energy. By speeding heat on its way, negentropic structures create order. Biological nets and ratchets that capture that negentropy are evolution's primary craft.

Positive feedback.

Positive feedback is the key exploiter and destroyer of Maxwell structures. The most impressive Maxwell structures are the ones in which the energy released in movement down the slope itself creates the conditions for further movement—where actions create chain reactions. A rolling stone gathers momentum, often sufficient to roll over or crush through other basins in its way. A spark can catch, with the energy released heating nearby tinder over their potential barriers, leading to fires, to conflagration, to firestorms. Hysterical words can set the neighbors a fighting; the frightened find safety in numbers, their kith rally 'round kin, reflecting their passion, and a local feud becomes a world war. The probability of contagion increases with the number of threats to a biological system, and otherwise latent pathogens are goaded into feeding-frenzy virulence when hosts become compromised.

Systems that dwell in Maxwell basins fascinate us as a match fascinates a boy; achieving 'maximum output with minimum input' seems an evolutionary imperative, leading to the technological virtuosity of *Homo sapiens*, and to the eventual perils of his chemical and nuclear fires. Enormous energy is required to fuse helium; but once carried over its Maxwell brim with a fission chain reaction, the fusion will shake the earth. Prometheus was the boy who taught the man how to trigger, burn, explode.

Negative

Negative feedback is the key defender of Maxwell structures, lending permanence to their otherwise fragile stability. A rock can "balance on a singular point of the mountain-side" because its base erodes more slowly under pressure, and thus, like a juggler on a geologic time scale, continuously corrects asymmetries to keep support under its center. The dissipative surface of combustion around a spark is large compared to its volume, and the flame usually gutters before kindling a great forest; the little spore



which would blight the potatoes must first tunnel through the potato's toxic skin. The little gemmule which makes us philosophers or idiots is crystalline DNA. It is seduced from its stable autonomy by recombination in meiosis, evolved to evade predators and repair coding errors, thereby defending the conjoint potential well of the species.

Exploiting Maxwell Structures

If we believe Maxwell that "All great results produced by human endeavor depend on taking advantage of these singularities", then it is near such singularities that the greatest potential for psychological endeavor are to be found. Some structures are remote from significant precipices, and thus possess little potential in their domain. Of those close to gradients, some reside at the bottom of deep potential wells, and rest adamantine; others, balanced for an instant at the tip of an unstable equilibrium, tumbled so quickly that their contemplation is reserved for the mind's eye, winked out in the early chaos of creation. How do we find malleable systems worth reengineering? How do we ease the scruples of a compulsive, or bind the lips that would utter words to set the world a fighting? How do we constrain bankers to be responsible; undermine corrupt and oppressive dictators; foster entrepreneurial societies? The answers are as different as the milieu in which the structures reside, but have some common principles.

Threats to stability

Complex systems are a resource for other complex systems, exploited wherever they are found. Predators on a system's negentropy are nature's value-added tax. They are themselves in turn dunned, having at each level their own species of fleas to bite them. Each therefore erects defenses to preserve the shallow well that guards their identity from incorporation into another's structure. Arms races between predator and prey are the first great engine of evolution, because life and sustenance are the preconditions for stability. Wheels need not be continually reinvented if their designs are passed on, whether in oral tradition or in DNA. Such investment drives the second great engine of evolution, the struggle to conserve those structures.

Stabilization is a counterproductive cost until stability is threatened, so the forces of stabilization are always reactions to agents of destabilization. Pounds of cure are more commonplace than ounces of prevention. Biological detectors uncover instabilities before they go over the top, rallying quenching mechanisms into play, be they at the level of antigens or grosser humors: Fears of strangers, of things that slither or go bump in the night, of pathogens, along with many other threats fingered by our paleo-control system, all goad our neo-control systems into defense. Firebreaks and quarantines stem contagion by separating compartments below their critical mass for kindling. Deviates are shunned, anarchists silenced, traitors hung, presses stopped. Long before Machiavelli governments mastered these techniques, and they are being constantly reinvented and redeployed (Wolf, 2007). Societies erect hurdles of tradition that only rebels surmount. Scruples are inculcated by moral preceptors, eased by psychotherapists, and exploited by populists. Disruption is channeled in time and space. Strategies of self-control work by bringing remote attractors to bear on behavior that is about to slip over into the pull of local gradients.

Stabilization that is too effective is also counterproductive, as it leaves the system in stasis. Biological systems become sitting ducks for predators. The "stability-plasticity dilemma" names the problem of finding the right balance of resistance to random change combined with a capacity for adaptive modification. Entrenched regimes often prevail until a novel threat topples their rigid defenses. Compared to clones, sexually reproducing organisms trade stability of (their individual) form and function for recombination with individuals that have at least comparable credentials for survival. Making selective modification sexy is perhaps the best way to effect change, but there are others.



Pushing

Maxwell structures can be destabilized by simply forcing the system over the brim. The efficiency of the technique depends upon the direction from which the force is applied. Pushing the system out of a multidimensional basin requires forces to be juggled—whether driving a recalcitrant herd, controlling nuclear fusion, or pushing wet spaghetti, pushing is plagued by problems of control and containment. Anger, an emergent, undifferentiated energy, can empower escape from the structure or destruction of its control mechanisms. Force often engenders reactance, and thus necessitates containment by forces of tradition, authority, or whip. Some guns in every war target the backs of compatriots to encourage them out of the trenches. Totalitarian techniques of crowd control—informers and spies and the threat of their ubiquity—were the enemies of the last century. Religious techniques of crowd control train the controlled to fashion and wear their own bridles and burkas—inculcating aversive moods of guilt, shame or sin to accompany every imagined liberty. Conjoined with state control, it is the enemy of our century. Contemporary Big Brothers watch from their windows, their computers, their heavens; they cause birds to fall to the ground, number the hairs on your head and the cookies in your computer. Escape is futile.

When the success of control is uncertain, controllers may push the system over its potential barrier strategically: Disease is precipitated by inoculation, avalanches by explosives, conspiracies by *agents* provocateurs, wars by preemptive strikes. Casus belli are uncovered or arranged. Object lessons are taught.

Pulling

Maxwell structures can be destabilized by pulling the system over its brim; where aversive control pushes, positive control pulls. Pulling systems out of minima requires a trace that will not break under the tension. It is difficult to engender long chains of behavior with distant reinforcers, and techniques of shaping, fading, and conditioned reinforcement are used to distribute the forcing energy along the proposed route. Leadership that transforms the trudge against the slope into an exhilarating climb is essential to all popular movements. The message of the leader must be echoed by the people and their presses, to keep it alive throughout the ascent. Progress is seen most often when each step finds some reward, through peers or publicists.

Momentum

Potential energy is transformed into kinetic energy as systems move downhill, and back into potential energy as they swing uphill again. The momentum of the system at the base of the slope is cashed out by motion uphill, again banked as increasing potential as the system mounts the slope. Systems without enough energy to overcome a potential barrier may store additional energy by moving back to get a running start, investing in an ascent of an opposite hill, and liberating that accrued potential in a downhill plunge. The energy for the ascent may be produced by converting energy from behind or in front. Flumes guide fluids downhill to rush over the next grade, pushed by those behind. Siphons constrain fluids down the next grade to suck more fluid up over the edge behind. Constraint by the pipes is essential. Heroic acts of the past inspire commitment from youth; once committed, they rush down the hills to find their own medals, their victory an inducement to the next generation. Inspiration by legends—saints, heroes and rebels—sustains most social endeavors.

Prodigious feats can be accomplished by organisms that are able to accumulate energy over longer epochs, and trigger it in an evanescent burst of power. Falcons ascend slow thermals to plunge explosively on prey; cicadas bend their ribs until they buckle, producing their piercing chirp; fleas compress a protein spring to release it, propelling leaps of hundreds of body-lengths. Humans often "get up a head of steam"



by methodically rehearsing past injuries or slights, energizing emotional states that carry them over obstacles. The elation of Easter wells up from the depression of Lent. War dances, tales of atrocities, and boot camps fortify armies to overcome hardship and manslaughter. Taboos restraining theft, rape, and murder are eroded in the fog of war. Politicians with their "fingers on the pulse" of their constituents personify movements already in progress, repudiating positions that are aging out of season. Such exploitations of momentum elicit the epithets "opportunist" or "leader", depending on one's own declination. The litany of the oppressed becomes the chorus of the militants, and martyrs become prudent investments toward the potential for breaking over.

Pumping

If there is a natural ebb and flow to movement within the basin, small additions of energy at each cycle may pump the system over the top, just as repeated pushes on a swing can raise it to giddy heights, and small inductions from a coil pump a laser beam through its mirrored crystal. Resonance stores energy dynamically, permitting amplification of precisely timed stimuli. Skillful confidence men dupe their victims by catching the rhythms of fear and greed, over-correcting on each cycle. Compensatory B processes following on the heels of A processes (Solomon and Corbit, 1973) are capitalized by the industries of addiction. When an individual's cycles of need and satisfaction are in phase with another's, the cumulative energy of that resonance has the potential to overwhelm all other control systems. Getting psyched, passionate love, and mass hysteria are some of the emergents of resonance. Individuals out of phase dampen each other's momentum. Downers, wet blankets, cold fish and realists modulate romantics, spastics, hyperactives, and enthusiasts—until they are ejected.

Saturation

The first task of a movement is to build a critical mass in space or time. Defenses can be overcome by continued or wide-spread assault. Springs in thermostats thrown for the *n*th time experience metal fatigue, just as parents saying "no" for the *n*th time experience mental fatigue. Systematic desensitization exhausts emotional barriers. A suitor may bring candy and flowers and dinners soft words and seclusion and gentle touch. Enough cannon fodder overwhelms entrenched positions. The civil rights movement sought to exhaust reactionary forces by saturating jails and courts with dissenters. Knowing this liability, many control systems deal harshly with wrong thinkers and repeat offenders in an attempt to stem the tide before it crests.

Erosion

Control systems may be worn down or suborned by denial of their legitimacy, by weakening their leverage, by eluding their control. It is wiser to disarm the enemy than fight him. War-paint and battle-dress cloak their warriors from moral responsibilities; press cards protect against charges of libel or voyeurism; tenure shields from political retaliation and efficiency review alike. The rich never break laws that they can bend or buy. Control is subverted by drugs that lull inhibitions, and by texts that undermine foundations. Because the denial of legitimacy is so fundamental a threat, it commands the full attention of the authorities; many jails in this world hold more political prisoners than civil offenders, just as the many hells of the next world contain more infidels than sinners.

Flanking

Passes through multidimensional landscapes may carry one out of a basin with less work than a frontal assault on the nearest potential barrier. These flanking maneuvers may work by appeal to elastic higher goods, such as consistency, liberty, or defense of God, with attraction to local signs of the goals



drawing the individual around the barrier to their ultimate realization. Finding the paths requires guides and gurus, rebels and heroes who are less constrained by local control mechanisms. Fear of the unknown is one of the greatest barriers, and once a path is found, others find it easier to follow: When Rosa Parks sat in the front of the bus from fatigue, it inspired many other blacks to sit in the front for the fight. Censoring deviant behavior and rumors of greener valleys are the first imperatives for all control systems. Blessed become the meek, the hungry, and the poor. All societies have formulated acts against sedition, indices of prohibited books, taboos, inquisitions, blacklists, and simple defamations for the unconforming. Tall stakes are hammered down. "Foreign" ideas are condemned generically; outlaws outlaw deviance, and saints are canonized only when they are safely in heaven, subject to interpretation. Control of the media is the first move of nascent tyrannies; a counter-control mechanism of some democracies is protection of the media.

Seduction

Insofar as the vast energy available from the passage down the slope can be brought to bear on behavior within the well, it may be used to motivate movement over the brink. But this must be done without engaging negative feedback. In humans, this is often accomplished by mislabeling—one is induced to attend a communist rally for intellectual reasons, or to be fair to both sides of the issue; to reveal military secrets in the cause of humanity; to visit his room to hear a new album; to consider the products of Madison Avenue as routes to status, economy, ease, modernity, youth, wisdom, and any destination other than hedonistic indulgence. Control mechanisms often focus the most opprobrium against the first wayward step. Once that step is taken, self-labeling—Born to be Wild, Rebel Without a Cause, Outsider—and the pleasures anticipated beyond the pale, make each additional step slipperier than the previous. Once enroute, dissonance is reduced—Why would I have been interested if there hadn't been some intrinsic value to the activity/product?—and consistency protected—How can I say 'no' now and save face?—leading many up the garden path until the gradient changes sign, and the die is cast.

Preserving Maxwell Structures

To realize the simplicity of objects at the top of Maxwell structures we must understand their code, as though they were glyphs to be deciphered. Without that key, we see but chaos and confusion. Structures are often invisible until they are understood; and too often we are ignorant that we are ignorant; we see not a mystery, but nothing. It was Newton's unweaving and reweaving of the rainbow that let us see light's dimensions for the first time. The mystery of the uncertain or the untamed provokes fear in the timid, curiosity in the brave. If only, those latter hope, I could break the code, find the grail, decipher the glyphs, then I shall find harmony, simplicity, truth. And perhaps also find a way to exploit the structure to my benefit. The drive for understanding—for discovering patterns of simplicity within the cacophony of the world, to ease fear or increase profit—is a universal. Many societies recognize the value of such curiosity by improving the perceptual skills of its citizens in schools and universities, and by treating the accumulation of knowledge as a primary good. Others squelch it as a threat to established order.

As important as the creation of knowledge is its preservation, in environments that are robust against dissolution; books that won't burn, bits that won't be wiped, Maxwell Structures that won't be eroded. Media that won't be left without readers. Evolution requires variation and selection and retention. Each of these processes—tolerance of variation, selection of productive innovation, and retention through access to secure information—form the core of stable societies. It is height of the elevation of the elite above the *hoi polloi* that energizes discontent. Equitable distribution of access to power—letting all who would attempt, reap the potential of knowledge—reduces the potential for strife. In the Federalist Papers and elsewhere, the founding fathers of the United States understood the importance of creating



one nation. The alternative, a federation of states, was unstable and would lead inevitably to internecine wars. They studied ways to implement positive and negative feedback, discovered the importance of a variety of time-constants, of seed operations, and the important different roles for self-quenching versus self propagating features; most importantly, they recognized the importance of a system in which self-interest would work toward stability, not against it. They designed the new democracy to last (Platt, 1966). If it is to last in nations that aspire to stable democracy, then citizens who teach citizens must keep this appreciation always before them, refine it, and teach it to their students. They must give them the keys to the code.

References

Georgescu-Roegen, N. (1971). The Entropy Law and the Economic Process. Cambridge, MA: Harvard University Press.

Hermeneutic circle. (2014, January 2). In Wikipedia, The Free Encyclopedia. Retrieved 21:16, March 2, 2014.

Jaynes, E. T. (1957). Information theory and statistical mechanics. Physical Review Letters, 106, 620-630.

Kelso, J. A. (1995). Dynamic Patterns: The Self-Organization of Brain and Behavior (Complex Adaptive Systems). Cambridge, MA: MIT Press.

Killeen, P. R. (1992). Mechanics of the animate. Journal of the Experimental Analysis of Behavior, 57, 429-463.

Maxwell, J. C. (1882) Science and free will. Reprinted in L. Campbell & W. Garnett (eds), *The life of James Clerk Maxwell*. London: Johnson Reprint Corp.

Platt, J. R. (1966). The step to man. New York: John Wiley & Sons, Inc.

Prigogine, I., & Stengers, I. (1984). Order out of chaos. New York: Bantam Books.

Schrödinger, E. (1944/1995). What is Life? Cambridge University Press.

Solomon, R. L., & Corbit, J. D. (1973). An opponent-process theory of motivation: II. Cigarette addiction. *Journal of Abnormal Psychology*, 81, 158.

Wolf, N. (2007). The end of America: Letter of warning to a young patriot. Chelsea Green Publishing, chelseagreen.com.